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Growth of a debonded void at a rigid secondary particle in a viscous metal

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When a ductile two-phase material is subjected to a high strain-rate deformation, the secondary particles nucleate voids, which will grow and coalesce, in a viscous matrix, leading to a dynamic ductile fracture. If the secondary particle is strong, the void nucleates at the matrix–particle interface and will grow without the shattering of the secondary particle. In this paper the growth of a debonded void at the secondary particle in a viscous metal has been studied theoretically in order to simulate the dynamic ductile fracture of two-phase materials. It has been assumed that the matrix is viscous and that the second-phase consists of randomly-dispersed rigid spherical particles. The analytical technique used in our study is a combination of the equivalent inclusion method of Eshelby and the back stress analysis method of Mori and Tanaka, by which the interaction between debonded voids are accounted for; hence the results presented are valid even for large volume-fractions of debonded voids. The theoretical results obtained in this study are compared with those for the case of complete voids nucleated by the shattering of weak particles.

1. Introduction

If the material of a structural element which is ductile is subjected to a high strain-rate deformation, the resulting failure mode will be dynamic ductile fracture. The ductile fracture of most commercial materials is due to secondary particles that nucleate voids, which subsequently grow and coalesce upon straining. The understanding of the mechanism of dynamic ductile fracture of materials is much less complete than that of brittle and quasi-plastic failure. It is understood, however, that there exist two kinds of modes for void nucleation: the shattering of weak particles and the debonding of the interface between the matrix and strong particles. The former case leads to the formation of a complete void, and the latter to that of a debonded one. It is also known that the ductile metals, such as copper and aluminium, subjected to a high strain-rate deformation, $\dot{\epsilon} = 10^4\ \text{sec}^{-1}$ or more, become viscous:

$$\sigma = \alpha + \beta\dot{\epsilon}, \tag{1}$$

where σ is the flow stress and α and β are constants for a certain range of $\dot{\epsilon}$ and flow temperature [1, 2].

Recently, Taya and Seidel [3] and Budiansky *et al.* [4] have theoretically investigated the growth of complete voids in a viscous metal. The interaction between voids, which will be enhanced as the volume-fraction of voids increases, has been taken into account in the work of Taya and Seidel [3]. However, Budiansky *et al.* concentrated their attention on an isolated void in an incompressible matrix; hence, the results obtained exaggerate the void growth for the case of uniaxial strain and lower it for the case of uniaxial tension, compared with those by Taya and Seidel [3].

In this paper the growth of a debonded void at the interface of a strong particle and a viscous matrix (e.g., $Cu-SiO_2$) under high strain-rate deformation is considered. In order to facilitate our computation it was assumed that the strong particle is spherical and rigid, and also that the matrix follows the behaviour of a viscous fluid under a constant strain-rate ($\sigma = \beta\dot{\epsilon}$) hence intertia effects could be neglected. The analytical technique

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used in this study is similar to that used in [3] i.e., the equivalent inclusion method of Eshelby [5] and the back stress analysis method of Mori and Tanaka [6]. According to the Mori–Tanaka method the interaction between debonded voids can be accounted for by an average back stress which can be evaluated in terms of eigen-strains [7].

A theoretical formulation of the growth of a debonded void is described in the next section.

2. Formulation of the theory

Consider an infinitely viscous body, D, which contains spherical, rigid particles of uniform size and which is subjected to uniaxial tension along the X_3-axis. Upon straining, the interface between the matrix and the strong particle is debonded and a debonded void will grow, as shown in Fig. 1. It is assumed that all of the debonded voids grow simultaneously, and that the matrix is both isotropic and incompressible. To make the applied stress a uniaxial tension, we apply the following strain-rate at infinity:

$$e_{33} = e_{\mathrm{A}}, \quad e_{11} = e_{22} = \nu e_{\mathrm{A}}, \tag{2}$$

where e_{A} is the prescribed constant strain-rate during the growth of debonded voids and ν is Poisson's ratio of the matrix which is set equal to 0.5. We will formulate the associated problem in a general form, i.e., without using $\nu = 0.5$, and then simplify the results by assuming incompressibility of the matrix.

Let the domain of the debonded voids be denoted by Ω and then the domain of the matrix becomes $D - \Omega$. The average of the stress disturbance due to all Ω is given in the matrix by [3, 6, 7]

$$\langle \sigma_{ij} \rangle_m = C_{ijkl} \tilde{e}_{kl}, \tag{3}$$

where $\langle \ \rangle$ denotes the volumetric average of a given quantity and C_{ijkl} is an isotropic viscosity tensor and is given by

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}). \tag{4}$$

The symbols λ and μ in Equation 4 correspond to the Lamé constants of an isotropic elastic material. $\tilde{e}_{kl}$ in Equation 3 is the average strain-rate in the matrix disturbed by all Ω. $\langle \sigma_{ij} \rangle_m$ and e_{ij} are unknown and will be determined later.

It is noted that for a debonded void enveloping a rigid spherical particle, the actual stress along the X_3-axis and strains along the X_1- and X_2-axes vanish; in Ω

$$\sigma_{33}^{\mathrm{A}} = 0 \tag{5}$$

and

$$e_{11}^{\mathrm{A}} = e_{22}^{\mathrm{A}} = 0, \tag{6}$$

where the superscript A denotes the actual quantity. Using the equivalent inclusion method of Eshelby [5–7], Equations 5 and 6 are reduced to, respectively,

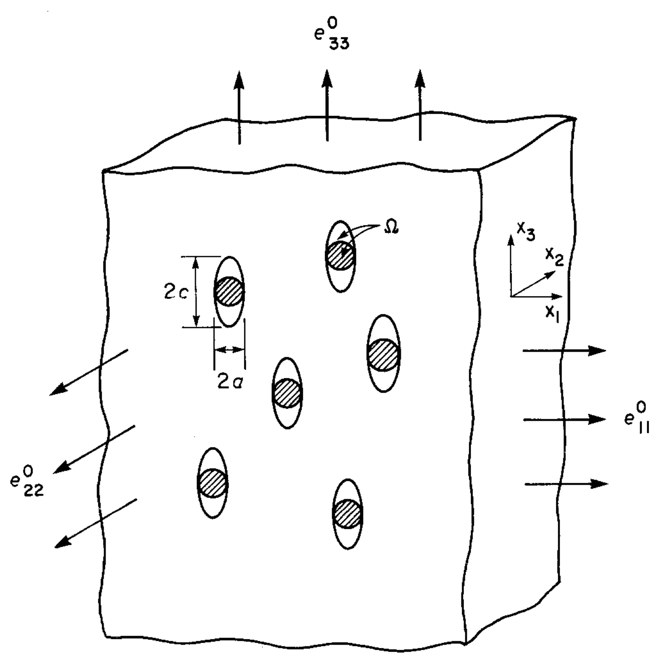

Figure 1 A theoretical model for the growth of a debonded void in a viscous metal subjected to the applied strain-rate, e^0_{ij}.

$$\sigma_{33}^{A} = C_{33kl}(e_{kl}^{0} + e_{kl} + \tilde{e}_{kl} - e_{kl}^{*}) = 0 \quad (7)$$

and

$$e_{11}^{A} = e_{11}^{0} + e_{11} + \tilde{e}_{11} = 0, \quad (8)$$

where e_{ij} is the strain-rate disturbance from introducing a single Ω into D, and e_{ij}^{*} is the eigenstrain defined only in Ω, i.e., $e_{ij}^{*} = 0$ in $D - \Omega$. If Ω is an ellipsoid, then e_{ij} is related to e_{ij}^{*} [5] by

$$e_{ij} = S_{ijkl}e_{kl}^{*} \quad (9)$$

where S_{ijkl} are the Eshelby tensors and they are a function of C_{ijkl} and the geometry of the ellipsoidal inhomogeneity, Ω. Since $\int_D \sigma_{ij}\,dv = 0$ [3, 6, 7], then

$$(1-f)\langle\sigma_{ij}\rangle_m + f\langle\sigma_{ij}\rangle_\Omega = 0, \quad (10)$$

where f is the volume-fraction of debonded void, and $\langle\sigma_{ij}\rangle_\Omega$ is given by

$$\langle\sigma_{ij}\rangle_\Omega = C_{ijkl}(e_{kl} + \tilde{e}_{kl} - e_{kl}^{*}). \quad (11)$$

In the derivation of Equation 11, the present authors have used

$$\sigma_{ij}^{0} = C_{ijkl}e_{kl}^{0}, \quad (12)$$

in D, where σ_{ij}^{0} is the uniform stress. The system shown in Fig. 1 gives rise to transverse isotropy and hence there are only two unknowns for each field variable (for example, e_{33}^{*} and $e_{11}^{*} = e_{22}^{*}$ for the eigenstrains). After having eliminated e_{ij} in the above equations by use of Equation 9, there are four unknowns, e_{11}^{*}, e_{33}^{*}, $\tilde{e}_{11}$ and $\tilde{e}_{33}$ which can be solved by use of Equations 7 to 10. The rate of the debonded void growth can then be expressed in terms of $\dot{c}/c$ as

$$\frac{\dot{c}}{c} = e_{33}^{0} + e_{33} + \tilde{e}_{33}, \quad (13)$$

where c is the length of the current major axis of the ellipsoid (see Fig. 1).

Having solved for e_{ij}^{*} and $\tilde{e}_{ij}$ and used Equations 9 and 13, then

$$\frac{\dot{c}}{c} = Qe_{A}, \quad (14)$$

where

$$Q = 1 + A_1S_{3333} + 2(1-f)A_2S_{3311} + f\left\{\frac{A_1}{(1-f)} - 2fA_3S_{3333} - \frac{2f^2}{(1-f)}A_3\right\}. \quad (15)$$

In Equation 15, the detailed expressions for A_1, A_2, A_3, S_{3333} and S_{3311} are given in the Appendix, after Poisson's ratio, $\nu = 0.5$, is substituted. Since Q changes with time t, Equation 14 is integrated incrementally to obtain

$$C_{i+1} = C_i \exp(Qe_A\Delta t), \quad (16)$$

where $\Delta t = t_{i+1} - t_i$ and the subscript i denotes the ith increment. It is noted that the length of the minor axis of the debonded void remains constant due to the rigidity of the spherical particle.

3. Results and discussion

In order to simulate the growth of a debonded void in a ductile metal deforming at a constant high strain-rate (for example $Cu-SiO_2$), the following data were used:

$$e_A = 10^4\ \text{sec}^{-1},$$

$$e_At_f = 0.4$$

and

$$t_f = 4 \times 10^{-5}\ \text{sec},$$

where t_f is the duration of the applied strain-rate e_A. Three values for the volume-fraction of debonded voids were used, $f = 0.05$, 0.1 and 0.3. 500 incremental steps were used to compute the current length of the void major axis, c. The growth of a debonded void is expressed in terms of $\ln(V/V_0)$, where V and V_0 are the void volume at the current and initial stages, respectively. The values of $\ln(V/V_0)$, plotted as a function of the applied strain e_At, are shown by the solid curves for the cases of $f = 0.05$, 0.1 and 0.3 in Fig. 2. Also in Fig. 2, values of $\ln(V/V_0)$ for the case of a complete void (without rigid particle) are plotted by the dotted curves [3]. It is clear from Fig. 2 that the growth of a debonded void is faster than that of a complete void. The volume-fraction of the second-phase particles, f, can be used as approximately that of the voided particles. Hence, it follows from Fig. 2 that the larger the volume-fraction of secondary particles, the faster the growth of a debonded void. The above conclusion can be also applied to the case of complete voids nucleated from the weak particles. The fact that the growth of a debonded void is faster than that of a complete void is due to the rigidity of the particles. To illustrate this, the natural tensile strains along the major [$\ln(c/c_0)$] and minor axes [$\ln(a/a_0)$] have been plotted in Fig. 3. The solid and dotted curves in Fig. 3 correspond to the cases of debonded voids and complete voids, respectively. The assumption that the strong particle is rigid yields $\ln(a/a_0) = 1$ at any level of straining, whereas $\ln(a/a_0)$ for the case

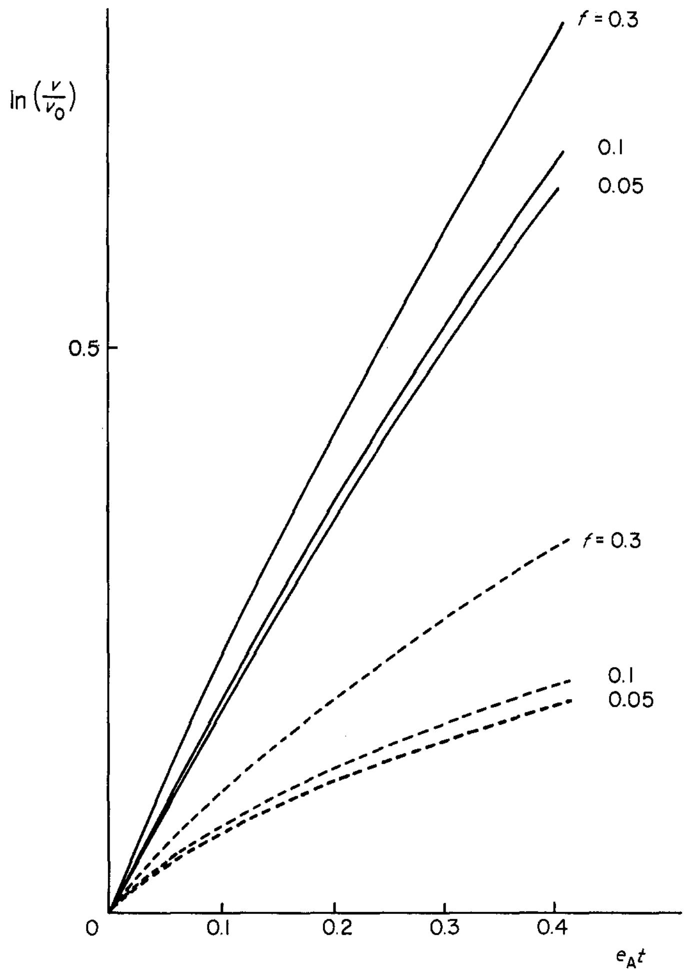

Figure 2 The logarithmic volume change $\ln(V/V_0)$ of a debonded (solid curves) and complete void (dotted curves) [3] against the applied strain $e_A t$. f is the volume-fraction of the voids.

of voids decreases as the applied strain $e_A t$ increases. A strong particle such as SiO_2 actually behaves elastically, but its transverse contraction along the X_1- and X_2-axes in Fig. 1 can be negligible compared with the longitudinal strain, $\ln(c/c_0)$, or the transverse strain $\ln(a/a_0)$, of a complete void.

The constant, α, in the stress–strain equation of a ductile metal deforming at high strain-rate, $\sigma = \alpha + \beta\dot{\epsilon}$, has no influence on the growth of debonded voids for the following reasons: the problem of the growth of debonded voids can be solved by combining the solutions of two cases, (a) an infinite body (without debonded void) subjected to uniaxial homogeneous tension along X_3-axis, α, and (b) an infinite body containing debonded voids, within which the actual stress is prescribed as $\sigma_{33}^{\mathrm{A}} = -\alpha$, and subjected to uniaxial tension. The second problem has been solved to obtain the expression of the void growth which has the same form as Equations 14 and 15 except that the terms carrying α in the expression of Q are multiplied by $(1-2\nu)$ and vanish upon the substitution of $\nu = 0.5$.

Next the validity of the present model is examined. For short deformation times (infinitesimal deformation), the present model gives rise to the compressive stress σ_{11}^{A} at the initial stage,

$$\frac{\sigma_{11}^{\mathrm{A}}}{T} = -\frac{3}{10}\frac{(1+5\nu)}{(1+\nu)}, \qquad (17)$$

where

$$T = 2(1+\nu)\mu e_{33}^{0}. \qquad (18)$$

Upon substitution of $\nu = 0.3$, we obtain $\sigma_{11}^{\mathrm{A}}/T = -0.577$, while the corresponding result obtained by Wang [8] is -0.378. The above discrepancy is due to the assumption in the present model that the void with a rigid particle is treated as a strong anisotropic inhomogeneity (Equation 5), and the contact region in the present model is smaller than that in [8]. Hence, the present model should not be used to predict the contact stress during the infinitesimal deformation of the void, but it should

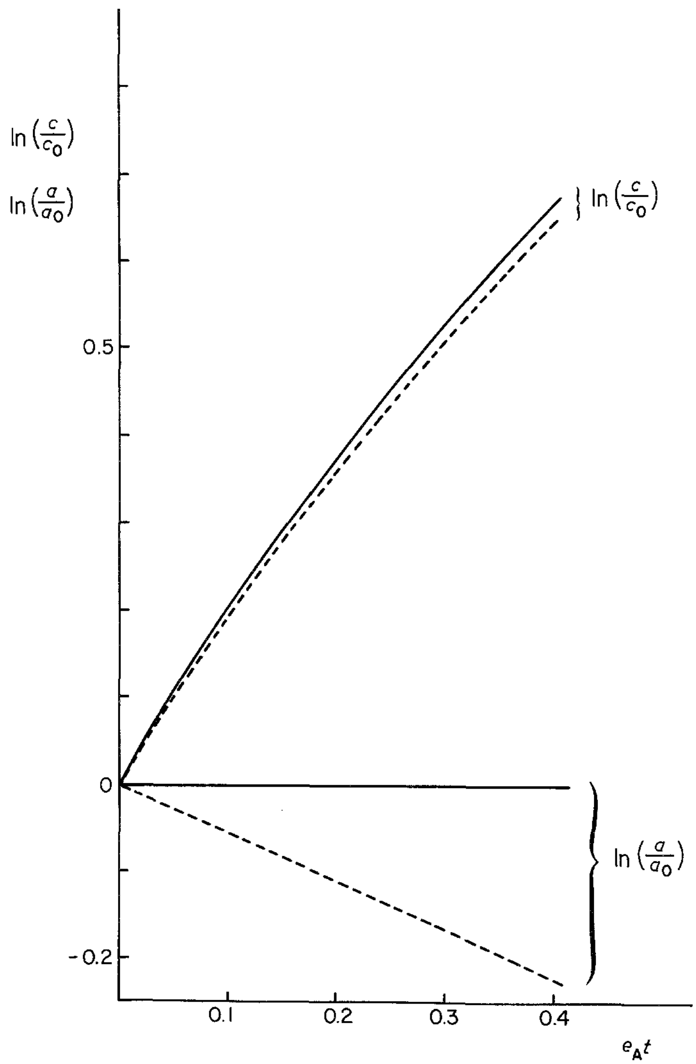

Figure 3 The natural strains along the major $[\ln(c/c_0)]$ and minor axes $[\ln(a/a_0)]$ of the void against the applied strain $e_A t$. The solid and dotted curves correspond to the cases of a debonded and a complete void for $f = 0.1$.

be focused on the finite deformation of the void, which without the present model would have been extremely difficult to solve. As far as the present authors are aware, no attempt has been made to measure experimentally the void growth of a two-phase metal under a very high strain-rate deformation with the state of uniaxial tension, except for the quasi-static case studied by Palmer and Smith [9]. Palmer and Smith observed the growth of a debonded void in Cu–SiO_2 under a quasi-static uniaxial tension. It is noted in [9] that the growth of the debonded voids near the fractured surface of Cu–SiO_2 is somewhat similar to that predicted in the present study, i.e., c/c_0, in Fig. 15 of [9] for the failure total strain 0.27, is about 2.0, whereas that predicted by the present model is 1.6. It is believed that the present results remain to be verified by a well-defined dynamic test where the applied strain-rate is kept as constant as possible.

4. Conclusions

The growth of debonded voids nucleated at rigid secondary particles in a viscous metal was studied theoretically in order to simulate the mechanism of a dynamic ductile fracture of a two-phase metal containing strong particles. The results are compared with those of a complete void nucleated at the weak particle, and have led to the following conclusions:

(a) The growth of the major axis of a debonded void, c, is slightly larger than that of a complete void, but the rate of the volume increase of a debonded void, $\ln(V/V_0)$ is much larger than that of a complete void.

(b) The larger the volume-fraction of the secondary particle, the larger the void growth becomes.

(c) The constant α in the matrix stress–strain-rate relation has no effect on the growth of a debonded void.

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Appendix: A_1, A_2, A_3, S_{3333} and S_{3311}

We will give below explicit expressions for A_1, A_2, A_3, S_{3333} and S_{3311} separately for spherical and ellipsoidal Ω. In the evaluation of A_1, A_2 and A_3, we express A_1, A_2 and A_3 formally in terms of S_{ijkl}, f and Poisson's ratio ν. Then, after having cancelled out the common factor, $(1-2\nu)$, in the numerator and denominator we substitute $\nu = 0.5$ to obtain the final results of A_1, A_2 and A_3.

A.1. Spherical Ω

$$A_1 = \tfrac{11}{10},$$

$$A_2 = \frac{21}{4(5+9f)}, \tag{A1}$$

$$A_3 = \tfrac{7}{2}A_2, \tag{A2}$$

$$S_{3333} = \tfrac{3}{5}$$

and

$$S_{3311} = -\tfrac{1}{5}.$$

A.2. Ellipsoidal Ω (the major axis along X_3-axis)

$$A_1 = \frac{2}{3I_0^2}\left(\frac{2\beta^2}{(\beta^2-1)}\left[1-\frac{3}{2}\frac{1}{(\beta^2-1)}\right]I_0\right), \tag{A3}$$

$$A_2 = \frac{([4\beta^2/(\beta^2-1)] + (2-[3\beta^2/(\beta^2-1)])I_0\}}{\left\{3(1-f)I_0^2 + 2f\left[-\frac{8\beta^2}{(\beta^2-1)} + \frac{2(4\beta^2-1)}{(\beta^2-1)}I_0\right]\right\}}, \tag{A4}$$

$$A_3 = \frac{[4\beta^2/(\beta^2-1)] + \{2-[3\beta^2/(\beta^2-1)]\}I_0)(-[4\beta^2/(\beta^2-1)] + \{1+[3/(\beta^2-1)]\}I_0)}{3I_0^2\left[\frac{3}{2}(1-f)I_0^2 + f\left\{-\frac{8\beta^2}{(\beta^2-1)} + \frac{2(4\beta^2-1)}{(\beta^2-1)}I_0\right\}\right]}, \tag{A5}$$

$$S_{3333} = 1 + \frac{2\beta^2}{(\beta^2-1)} - \frac{3}{2}\frac{\beta^2}{(\beta^2-1)}I_0 \tag{A6}$$

and

$$S_{3311} = \frac{1}{(\beta^2-1)} + \frac{3}{4}\frac{1}{(\beta^2-1)}I_0, \tag{A7}$$

where

$$I_0 = \frac{2\beta}{(\beta^2-1)^{3/2}}(\beta(\beta^2-1)^{1/2} - \cosh^{-1}\beta) \tag{A8}$$

and

$$\beta = \frac{c}{a}. \tag{A9}$$

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